Spectral Statistics of Random Matrices and Numerical PDEs

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Abstract

We study connections between the spectral distributions of random matrices and discretizations of elliptic partial differential operators. By examining eigenvalue statistics of finite-difference Laplacians perturbed by random noise, we demonstrate convergence toward universal ensembles predicted by random matrix theory. This provides a computational framework linking numerical PDE solvers to statistical predictions from quantum chaos.

1 Introduction

Random matrix theory (RMT) has long described universal spectral features across quantum systems, number theory, and high-dimensional statistics. In numerical PDEs, discretizations of operators such as the Laplacian produce large structured matrices whose spectra encode analytic properties of the underlying domain.

2 Main Observation

Let L_h denote the finite-difference Laplacian on a uniform grid of mesh size h, and let $A_h = L_h + \epsilon R_h$, where R_h is a random symmetric perturbation. As $h \to 0$ and $\epsilon \to 0$, the normalized eigenvalue gaps of A_h converge in distribution to those of the Gaussian Orthogonal Ensemble (GOE).

3 Numerical Illustration

Simulations of A_h on $[0, 1]^2$ using 100×100 grids reveal that even small random perturbations produce GOE-like level spacing statistics. This confirms the universality conjecture in a numerical PDE setting.

4 Conclusion

The spectral universality observed in numerical PDE matrices suggests a deep interplay between discretization theory and random matrix models, with implications for stability analysis and uncertainty quantification.

References

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